16-Term Error Model and Calibration Procedure for On-Wafer Network Analysis Measurements

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Abstract—**Vector network measurements are enhanced by calibrating the measurement system over the entire band of interest. This is presently done using a 12-term error correction model. Many measurement systems including open air devices, such as MMIC wafer probes, contain leakage and coupling error terms not modeled in current calibration systems. In this paper all error terms in such a system are included in a new 16-term error model and calibration procedure. Corrected measurements using the new 16-term calibration procedure are compared with TRL and 12-term calibration measurements and excellent agreement is observed for a non-leaky system. For a leaky system, the 12-term model is shown to break down while the 16-term model retains its accuracy. These results validate the accuracy and viability of the new calibration procedure for MMIC wafer probe measurements and other measurement systems containing leakage.**

I. Introduction

THE ACCURACY and usefulness of the Vector Network Analyzer (VNA) is enhanced by calibrating the measurement system at the device under test (DUT) interface. The calibration should be capable of providing a repeatable representation of the measurement system and account for most of the system errors. A large number and variety of error models and calibration procedures have been proposed to date [1]. These include the 12-term error model [2], TRL [3], TSD [4] and others [5]. Although these models are accurate for many measurement systems, they include only a portion of the possible errors in a measurement system and do not include many of the leakage and coupling terms often encountered in MMIC measurements.

In an MMIC wafer probing system, the wafer probes utilize open air fixtures which result in leakage and coupling errors not modeled and accounted for in the 12-term or other models. Consequently, if accurate MMIC measurements are expected, it is necessary to include these new leakage terms in the error model.

It is the purpose of this paper to present an error model and a new calibration procedure which accounts for all of the errors in an open air fixture such as an MMIC device; in the case of a two-port network, this extends to 16 terms. The 16-term model will allow fixtures that have poor grounding and numerous cross-talk paths to be accurately calibrated. This paper will investigate the theory and methods used to solve the 16-term error model system as well as simulation and measurement results obtained from this model.

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II. General Theory

For a two-port measurement such as that shown in Fig. 1, the 16-term S-parameter error model is shown in Fig. 2. As shown the dotted arrow terms represent the cross-talk paths of which six terms are not accounted for in the conventional 12-term error model. Using flow-graph analysis, the error adapter is represented in matrix form as

$$\begin{bmatrix} b_0 \\ b_3 \\ b_1 \\ b_2 \end{bmatrix} = E \begin{bmatrix} a_0 \\ a_3 \\ a_1 \\ a_2 \end{bmatrix},$$

$$E \triangleq \begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix} = \left[\begin{array}{cc|cc} e_{00} & e_{03} & e_{01} & e_{02} \\ e_{30} & e_{33} & e_{31} & e_{32} \\ \hline e_{10} & e_{13} & e_{11} & e_{12} \\ e_{20} & e_{23} & e_{21} & e_{22} \end{array}\right]. \quad (1)$$

The measured (S_m) and the actual (S_a) S-parameters of the device under test are defined as

$$\begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = S_m \begin{bmatrix} a_0 \\ a_3 \end{bmatrix}, \qquad S_m = \begin{bmatrix} S_{11m} & S_{12m} \\ S_{21m} & S_{22m} \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = S_a \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \qquad S_a = \begin{bmatrix} S_{11a} & S_{12a} \\ S_{21a} & S_{22a} \end{bmatrix}. \quad (3)$$

The objective is to establish a calibration procedure that may be used to calculate all the error terms in the matrix E so that the actual matrix S_a of the device under test may be extracted from the measured S-parameters S_m.

By applying the definitions of S_m and S_a to (1) and applying linear algebra operations, a relationship between

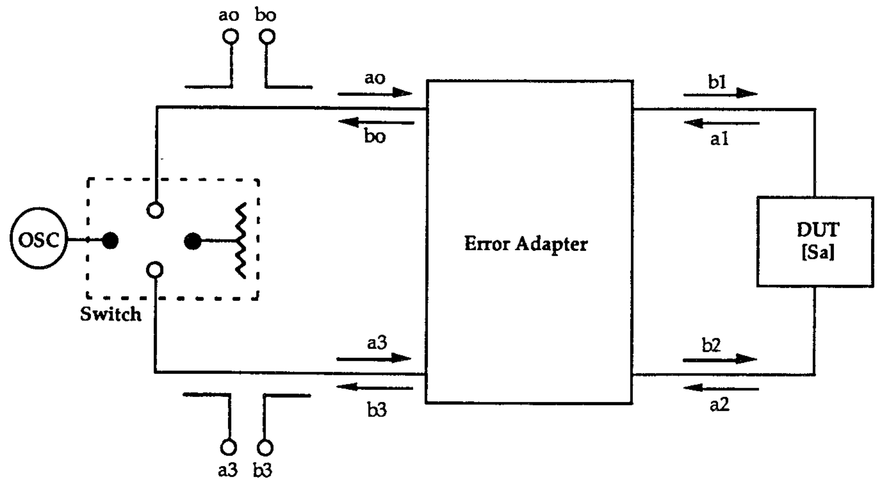

Fig. 1. Block diagram of a two-port system.

S_a, S_m and the error matrix $\boldsymbol{E}$ is obtained as (4)

$$S_m = E_1 + E_2 S_a (I - E_4 S_a)^{-1} E_3 \tag{4}$$

where $\boldsymbol{I}$ is the unit matrix.

Solving for S_a, we obtain

$$S_a = \{E_3 (S_m - E_1)^{-1} E_2 + E_4\}^{-1}. \tag{5}$$

Detailed expansion of (5) shows that it is nonlinear in the error terms thus making it difficult to solve for the error terms in terms of the calibration standards.

On the other hand, by using cascading T-parameters to represent the error terms, a linear set of equations and an easier solution may be obtained. The error system of (1) may be represented in terms of T-parameters as

$$\begin{bmatrix} b_0 \\ b_3 \\ a_0 \\ a_3 \end{bmatrix} = T \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix},$$

$$T \triangleq \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} = \left[\begin{array}{cc|cc} t_0 & t_1 & t_4 & t_5 \\ t_2 & t_3 & t_6 & t_7 \\ \hline t_8 & t_9 & t_{12} & t_{13} \\ t_{10} & t_{11} & t_{14} & t_{15} \end{array}\right]. \tag{6}$$

By applying the definitions of S_m and S_a to (6) and applying linear operations, the following relationships between S_m, S_a and $\boldsymbol{T}$ are obtained:

$$S_m = (T_1 S_a + T_2)(T_3 S_a + T_4)^{-1} \tag{7}$$

$$T_1 S_a + T_2 - S_m T_3 S_a - S_m T_4 = 0 \tag{8}$$

$$S_a = (T_1 - S_m T_3)^{-1} (S_m T_4 - T_2). \tag{9}$$

Note that (8) is a set of four homogeneous equations that are linear in the entries of the partitioned $\boldsymbol{T}$ matrix. Theoretically, by using four different two-port standards, enough equations are generated to solve the 16 $\boldsymbol{T}$ error terms. Once $\boldsymbol{T}$ is solved, equation (9) can then be used to determine the actual S-parameters S_a of an unknown DUT from its measured S-parameters S_m. The developed 16-term model is general and because it takes into account the leakage terms, it improves accuracy for measurement systems containing leakage.

III. Solving the T Error Terms

Once the set of equations is obtained, the problem lies in solving the system of homogeneous equations $\boldsymbol{A} \cdot \boldsymbol{T} = 0$. Other calibration methods have been suggested to date which use cascading error networks [4], [5]. Most have taken special cases and have introduced assumptions to simplify the mathematical manipulations required to solve for the error network. Along with the theory, this paper introduces a general, accurate method for solving $\boldsymbol{A} \cdot \boldsymbol{T} = 0$ for the error terms of a network analyzer.

A. Solution Method

The trivial solution $\boldsymbol{T} = 0$ is always a possible solution for a homogeneous set of equations. In fact if $\boldsymbol{A} \neq 0$, then the matrix equation $\boldsymbol{A} \cdot \boldsymbol{T} = 0$ has only the trivial solution $\boldsymbol{T} = 0$. Therefore the solution for $\boldsymbol{T}$ is nontrivial if and only if $|\boldsymbol{A}| = 0$ [6].

In this situation, the matrix equation generated by the 16-term error model contains two singularities. This is due to the inability to measure independently the e_{10} and e_{23} error terms. In the 12-term model these terms are not solved for independently but in combination with e_{01} and e_{32} [1]. Since the T-terms represent the E-terms, the same singularities exist in both matrices, although they are not as obvious in the $\boldsymbol{T}$ matrix.

The homogeneous system of 16 equations can be solved, in principle, by a variety of methods. The equations may be normalized to one of the unknown coefficients, preferably one whose magnitude is close to unity, yielding an equation of the form $\boldsymbol{A} \cdot \boldsymbol{T} = \boldsymbol{B}$, and solving the resulting 15 equations using one of the routine direct solution methods. Alternatively, the over-determined system of 16 equations may be solved in a least-squares sense using a procedure such as the singular value decomposition (SVD) method [7]. Initial experience with the first procedure

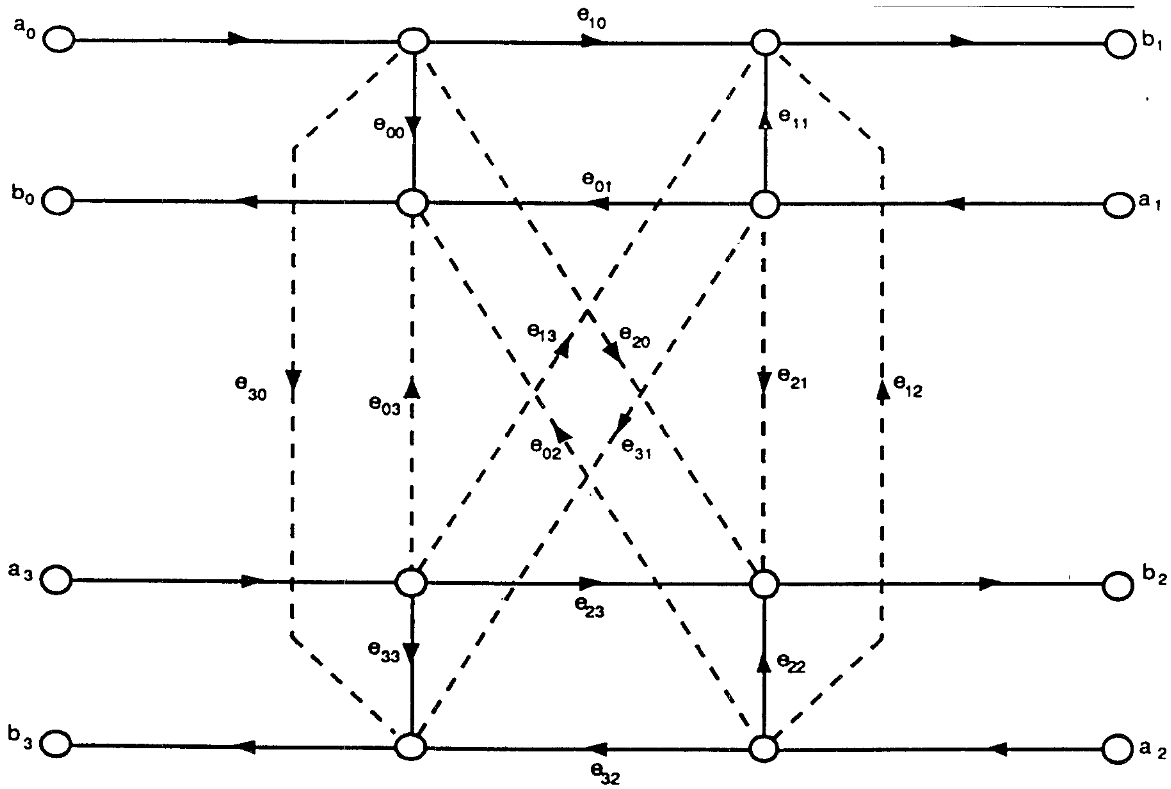

Fig. 2. Flow graph of the 16-term error adapter corresponding to the system shown in Fig. 1.

provided unsatisfactory results probably due to the singularities and a successful solution was obtained using SVD.

B. Solving Least Squares Using Singular Value Decomposition

Singular value decomposition (SVD) was used to solve the least squares problem for the 16 terms of the error model. SVD was chosen for three reasons. First, SVD is designed to handle singularities. Second, in an overdetermined system, which is the case here, SVD produces a solution that is the best approximation in the least-squares sense. The final and most important reason is that SVD provides additional valuable information about the system. This includes the condition of the matrix, singularities and an indication of noise and other systematic errors present in the system of equations. Although other methods may be faster, the information from SVD proves to be very valuable in understanding and solving many of the problems associated with the singularities and systematic errors in the 16-term system. A detailed discussion of the mathematics of SVD can be found in the literature [7], [8].

The basis of SVD is that any $m \times n$ matrix A can be decomposed into an equivalent product of three matrices $\boldsymbol{U}$, $\boldsymbol{W}$, and $\boldsymbol{V}^T$ where $\boldsymbol{U}$ is a column-orthogonal matrix, $\boldsymbol{W}$ a diagonal matrix and $\boldsymbol{V}$ a row-orthogonal matrix.

$$A = U \cdot \left[\operatorname{diag}(w_j)\right] \cdot V^T. \tag{10}$$

SVD performs this decomposition of A regardless of the singularities of A. The number of singularities of A is determined by the number of zero w_j elements. The condition number of A is defined as the ration of the largest w_j to the smallest w_j. In a noiseless system, the smallest w_j is identically zero. As noise is added to the system a corresponding increase in the smallest w_j results. In simulation and noise analysis it was determined that the value of the smallest w_j corresponded approximately to the level of noise and systematic errors in the system. For instance, if noise at a level of -60 dB is added to the system, then the smallest w_j will be approximately -60 dB. Thus SVD not only solves the system equations but also gives additional valuable information about the system and a good indication of the validity of the solution.

Once SVD is performed on A, the solution to the $\boldsymbol{T}$ matrix can be obtained [7]:

$$T = V \cdot \left[\operatorname{diag}(1/w_j)\right] \cdot U^T \cdot B \tag{11}$$

where $\boldsymbol{B}$ is a column vector obtained from the normalization of the homogeneous set of equations (8).

IV. Simulation and Measurement Results

A. Measurements of a Non-Leaky Coaxial System

For verification of the 16-term model theory and calibration procedure, the model was first simulated and then implemented on the HP8510B network analyzer via HPIB communication to an external controller. First, extensive simulations were performed to verify the accuracy of the new procedure. Then a 51 point calibration was performed on the HP8510B from 2 to 12 GHz on a 7 mm

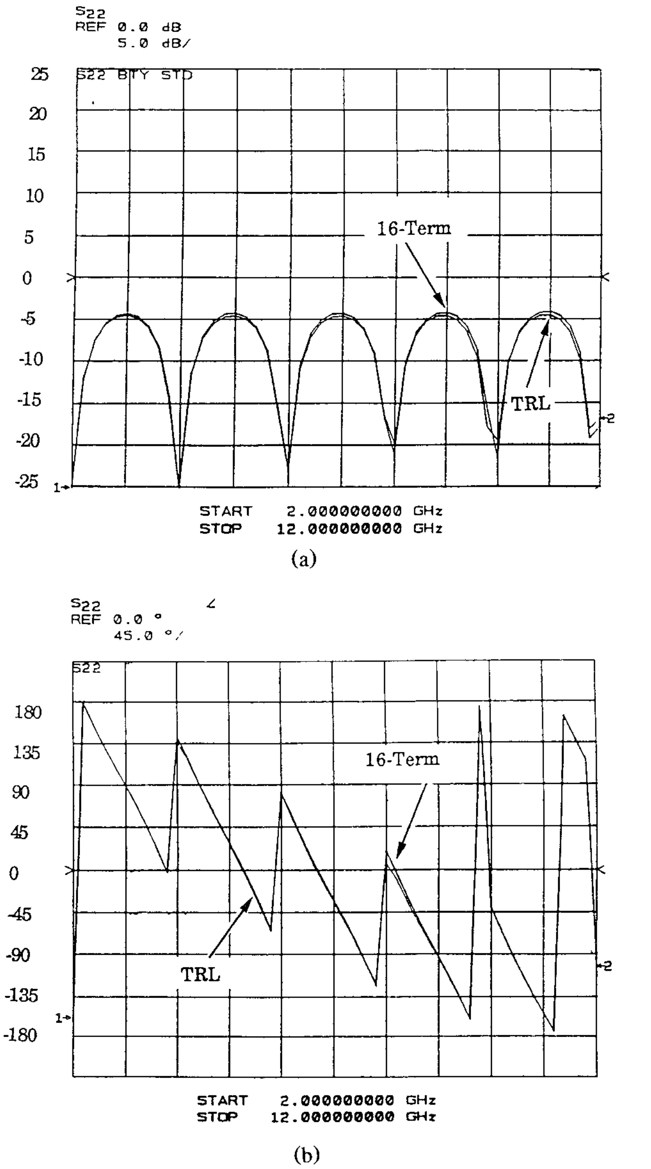

Fig. 3. Comparison of measurement of a 25 Ω mismatch airline using the 16-term calibration procedure and a TRL calibration of a 12-term model. (a) Magnitude. (b) Phase.

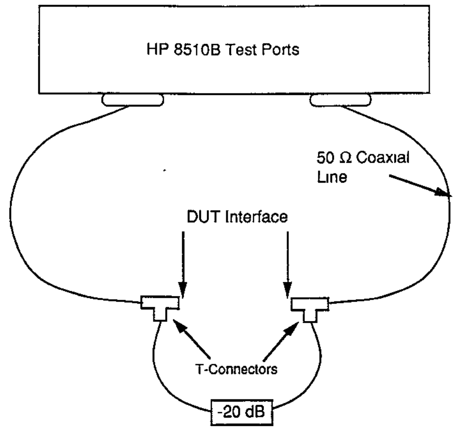

Fig. 4. Measurement configuration to simulate leakage at the DUT ports.

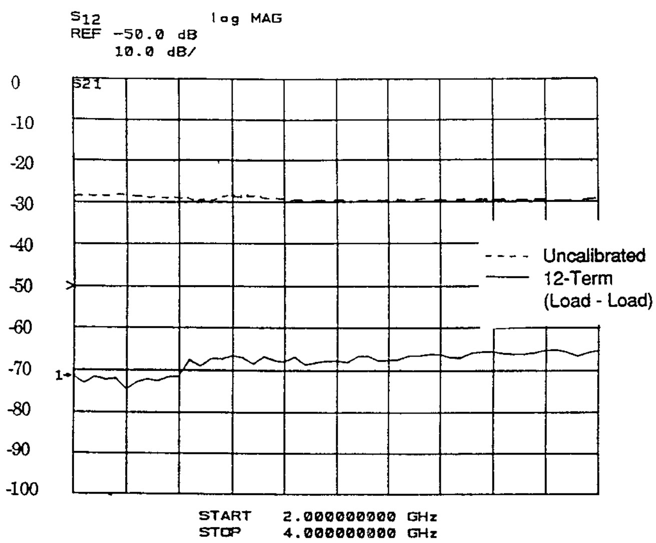

Fig. 5. Isolation measurements (S_{12}) comparing uncorrected and after 12-term correction with matched loads at the DUT ports.

coaxial measurement system. Two different verification standards, a 20dB attenuator and a mismatch airline, were used. The corrected measurements obtained from the 16-term calibration were then compared to measurements made using a 12-term, TRL calibration of the same measurement system.

The 16-term measurements corresponded very well to the 12-term, TRL-calibrated measurements, as shown in Fig. 3. The phase measurements can hardly be distinguished in Fig. 3(b). It thus confirms the validity of the 16-term error model system and calibration procedure. There was, however, a slight difference in the measurements. This difference is most likely due to imperfections in the calibration standards used. Unlike TRL, since all four standards need to be fully known in the 16-term model calibration procedure, the calibration standards must match their models for accurate results. The four standard combinations used in this calibration were THRU, MATCH-MATCH, OPEN-SHORT, and SHORT-OPEN. The difference in measurements is then about the same as the difference between 12-term calibrations using TRL and using OPEN/SHORT/LOAD/THRU. Another source of error can be attributed to the fact that in a coaxial measurement system many of the leakage terms are below the noise and systematic error levels in the system. It is common practice, in such cases, to set the leakage terms to zero which yields better results than averaged measurements of noise. By including these terms in the calculations, another source of systematic error is introduced.

B. Measurements of a Leaky System

The measurement configuration shown in Fig. 4 was used to provide a very simple approximation to a leaky system such as an MMIC wafer probing station. As can be seen in the figure, this measurement system provides additional leakage paths which are not included in the

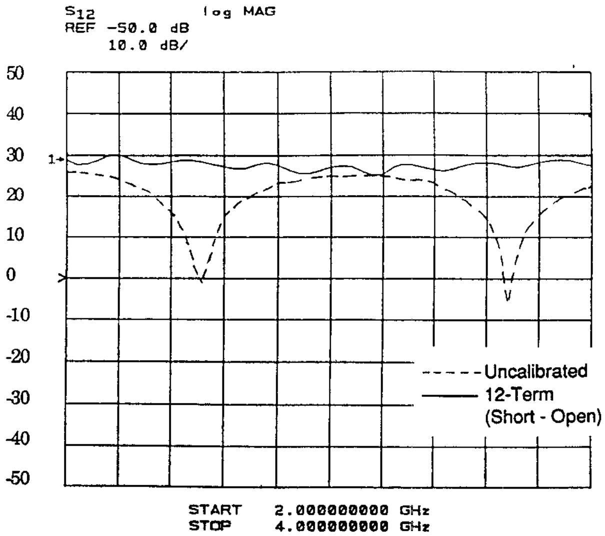

Fig. 6. Isolation measurement (S_{12}) comparing uncorrected and after 12-term correction with short and open connected at the DUT ports.

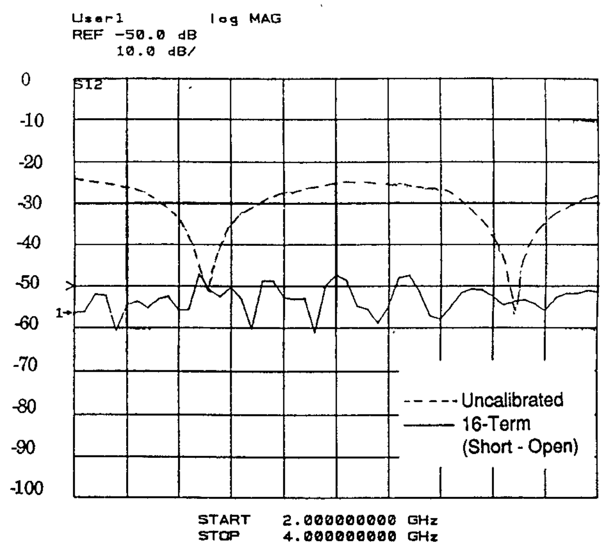

Fig. 8. Isolation measurement (S_{12}) comparing uncorrected and after 16-term correction with short and open connected at the DUT ports.

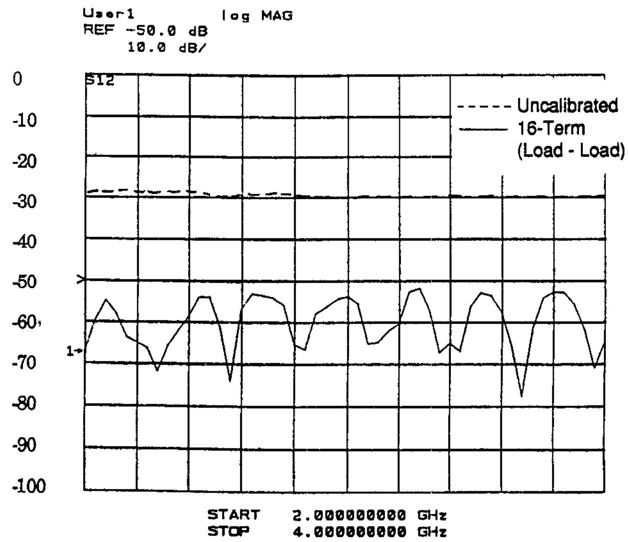

Fig. 7. Isolation measurement (S_{12}) comparing uncorrected and after 16-term correction with matched loads connected at the DUT ports.

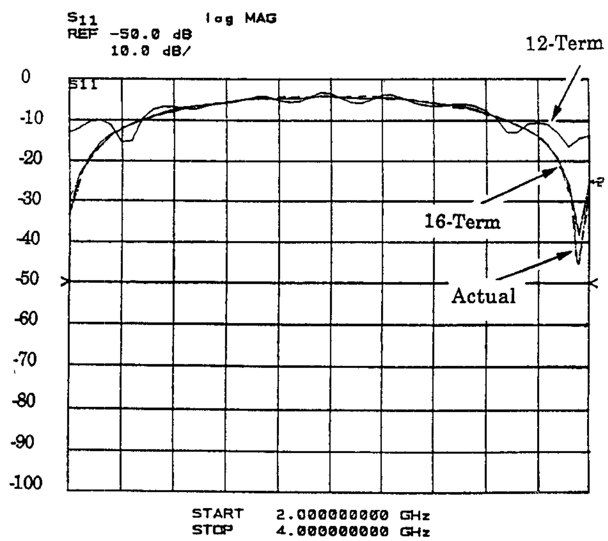

Fig. 9. Measurements of a 25 Ω mismatch airline comparing ideal (12-term model with no leakage) measurements to the 12-term and 16-term error models in a leaky system.

12-term model. However, the 16-term error model does include these paths. It will be shown that the performance of the 12-term model is significantly reduced while the performance of the 16-term calibration remains good.

For this measurement system, a 12-term OPEN/SHORT/LOAD/THRU calibration (including isolation) was performed along with the 16-term calibration. The performance of the 12-term model when measuring isolation (Fig. 5) where the test ports are terminated by matched loads is very good (≈ -65 dB). It is clear that the 12-term model fails to isolate the leakage paths when measuring a highly reflective device as can be seen from Fig. 6. When shorts or opens are connected at the test ports, the isolation measurement increases to approximately that of the leakage path (≈ -20 dB). Ideally, the isolation measurements should not change when different DUT's are measured. This illustrates one situation where the 12-term model does not work.

On the other hand, the 16-term isolation measurements remain consistently good (≈ -60 dB) for both highly reflective and well matched termination devices. This is illustrated in Figs. 7 and 8 and the results obtained clearly demonstrate the advantage of the 16-term calibration procedure when measuring a leaky system.

A mismatch airline was also measured with both 12-term and 16-term procedures as shown in Fig. 9. As can be seen, the 16-term measurements are much better than the corresponding 12-term measurements by comparison with the measurement of a non-leaky system, shown in Fig. 3(a), over the range 2 to 4 GHz. Again, this shows the advantage of using the 16-term calibration procedure over that of the 12-term procedure when measuring with a leaky system such as an MMIC wafer probe station.

V. Conclusion

The 16-term error model and calibration procedure has been successfully developed, simulated and implemented using the HP8510B. It provides a general model and method for characterizing all the errors in a four-port error adapter. Good results were obtained in both simulation and measurements showing the validity of the 16-term error model. The 16-term calibration was also made on a leaky measurement system emulating the situation of a wafer probing station. Measurements were made using T-junctions in each of the two ports to facilitate simulation of coupling between the ports.

This work has demonstrated that the leakage effects can be measured and corrected. However, the on-wafer measurement environment has additional error, such as probe placement, location and repeatability, wafer motion and perturbations caused by the presence of the device under test, all of which will change the leakage paths.

After calibration, the 16-term model performed significantly better than the corresponding 12-term model of the same leaky system. The results demonstrate an improvement over the 12-term model, taking into account the leakage terms and hold the potential for providing for more accurate network analysis of MMIC measurement devices.

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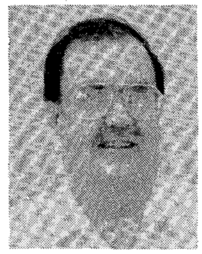

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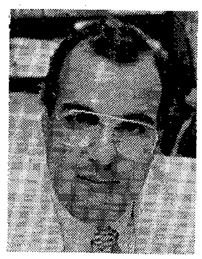
He joined the Hewlett-Packard Company in 1966 and has worked on virtually every microwave network analyzer product introduced since then. Since 1972 he has been R&D Section Manager for High Performance Network Analyzers at the Network Measurements Division of Hewlett-Packard in Santa Rosa, CA. He is heavily involved in error correction techniques and measurement accuracy analysis in the microwave industry.



Magdy F. Iskander (S'72–M'76–SM'84) was born in Alexandria, Egypt, on August 6, 1946. He received the B.Sc degree in electrical engineering from the University of Alexandria, Egypt, in 1969, and the M.Sc. and Ph.D. degrees both in microwaves, from the University of Manitoba, Winnipeg, MB, Canada, in 1972 and 1976, respectively. In 1976, he was awarded a National Research Council of Canada Postdoctoral Fellowship at the University of Manitoba.

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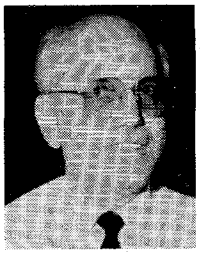
Dr. Iskander edited two special issues of the *Journal of Microwave Power*, one on "Electromagnetics and Energy Applications," March 1983, and the other on "Electromagnetic Techniques in Medical Diagnosis and Imaging," September 1983. The holder of seven patents, he has contributed seven chapters to five research books, published more than 90 papers in technical journals, and made more than 150 presentations at technical conferences. In 1983 he received the College of Engineering Outstanding Teaching Award and the College Patent Award for creative, innovative, and practical invention. In 1984, he was selected by the Utah Section of the IEEE as the Engineer of the Year. In 1984 he received the Outstanding Paper Award from the International Microwave Power Institute, and in 1985 he received the Curtis W. McGraw ASEE National Research Award for outstanding early achievements by a university faculty member. In 1991 he received the George Westinghouse Award for innovation in Engineering Education. In 1986 he established the Engineering Clinic Program in the College of Engineering at the University of Utah. Since then the program has attracted more than 30 research projects from 14 different companies throughout the United States. He is also the director of the NSF/IEEE Center for Computer Applications in Electromagnetics Education (CAEME). He was one of the organizers of a Symposium on "Microwave Processing of Materials," held in conjunction with Materials Research Society, Spring of 1990 and 1992, in San Francisco.



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Marc Vanden Bossche, photograph and biography not available at the time of publication.